An Analysis of Wideband Direction-of-Arrival Estimation for Closely-Spaced Sources in the Presence of Array Model Errors

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***Abstract*—This letter presents an analysis of wideband direction-of-arrival (DOA) estimation for closely-spaced sources using arbitrary antenna array taking into account the effect of array model error, which is important issue in practical implementation. Based on this analysis, a new wideband DOA estimation method without array calibration is then developed to deal with the effects of array errors. The performance improvement of the proposed method in the presence of array errors is shown in simulation results.**

***Index Terms*—Array model errors, DOA estimation, smart antennas.**

I. Introduction

FUTURE generation of wireless communication systems will use smart antennas to improve the performance and the spectrum efficiency of the system [1]. Moreover, in future systems, wideband signals will be used for requirements of higher data rate services. In this letter the direction-finding smart antennas approach is considered [1]. This approach has two phases: first, the directions of users are estimated during reception (DOA estimation) and then the direction information is used to calculate the weights for array transmission (beamforming) [2]. In this letter, the wideband DOA estimation for wideband smart antennas in future wireless communication systems is considered.

Most of the results for DOA estimation have been obtained for narrowband signals [3], [4] and/or via simulations. In [5], by using Bessel functions the source-direction vector is separated into two components: one depends on frequency and array parameters, and the other depends only on directions of arrival. The Taylor series can be applied to describe the dependence of the latter component on the source separation. The behavior of closely-spaced sources can be then analyzed by considering the source-direction vector when the source separation goes to zero. However, in [5], the effects of array model errors (element position error, element gain and phase errors) are not yet considered.

In this letter, an extension of the analysis in [5] with taking into account the array model errors is presented. In the effective wideband DOA estimation methods introduced in [7]–[12], the array is assumed to be ideal, it means that the positions and the electrical characteristics (gains and phases) of antenna elements are accurate. However, in practice, especially for wideband signals and at millimeter-wave band, these array assumptions are not much satisfied due to the uncertainties of element positions and fluctuations of element gains and phases (due to nonidentical elements, frequency-variant response, aging, environmental conditions etc.). Therefore, unlike the ideal case, the (actual) source-direction vectors at different frequencies within the signal bandwidth will consist of known components (nominal) and unknown ones (perturbations or errors). After deriving the dependence of source-direction vector on (actual) array parameters and the source separation, a new wideband DOA estimation method without array calibration, *Modified Nominal Transformation (MNT)*, is proposed to overcome these errors.

II. Wideband DOA Estimation in the Presence of Array Model Errors

A. Wideband Model in the Presence of Array Model Errors

Consider an arbitrary antenna array composed of M elements, all elements are assumed to be omnidirectional and mutual coupling between elements is not taken into account. Assumed that P wideband signal sources, with identical bandwidth B, are located in the far-field of the array, impinging on the array from distinct directions $\Omega_p = (\theta_p, \phi_p)$, $p = 1 \ldots P$, where θ_p and ϕ_p are elevation and azimuth, respectively, $0 \leq \theta_p \leq \pi/2$, $-\pi \leq \phi_p \leq \pi$. Each signal has temporal frequencies ω_k, which are observed over time interval of T, $\omega_k = 2\pi k/T$, $\omega_l \leq \omega_k \leq \omega_h$, where ω_l and ω_h are lowest and highest frequencies, respectively, included in the bandwidth B. From [6], the ($M \times 1$)-vector of signals at outputs of the array in frequency-domain, $\mathbf{X}(\omega_k)$, is expressed by

$$\mathbf{X}(\omega_k) = \mathbf{A}(\omega_k)\,\mathbf{S}(\omega_k) + \mathbf{N}(\omega_k) \tag{1}$$

where $\mathbf{S}(\omega_k)$ is ($P \times 1$)-vector of signals at the input of array, $\mathbf{N}(\omega_k)$ is ($M \times 1$)-vectors of white noise at the array, and

$$\mathbf{A}(\omega_k) = [\mathbf{a}(\omega_k, \Omega_1), \ldots, \mathbf{a}(\omega_k, \Omega_p), \ldots, \mathbf{a}(\omega_k, \Omega_P)] \tag{2}$$

is the ($M \times P$)-actual source-direction matrix, with actual source-direction vector, $\mathbf{a}(\omega_k, \Omega_p)$, given by

$$\begin{aligned}\mathbf{a}(\omega_k, \Omega_p) = \Big[&\gamma_1(\omega_k) e^{j\psi_1(\omega_k)} e^{-j\omega_k \tau_1(\Omega_p)}, \ldots,\\ &\ldots, \gamma_m(\omega_k) e^{j\psi_m(\omega_k)} e^{-j\omega_k \tau_m(\Omega_p)}, \ldots,\\ &\ldots, \gamma_M(\omega_k) e^{j\psi_M(\omega_k)} e^{-j\omega_k \tau_M(\Omega_p)}\Big]^T\end{aligned} \tag{3}$$

Manuscript received February 4, 2003; revised April 3, 2003. The review of this letter was arranged by Guest Editor Roberto Sorrentino.

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Digital Object Identifier 10.1109/LMWC.2003.815701

where $\gamma_m(\omega_k)$ and $\psi_m(\omega_k)$ are actual gain and actual phase of mth element, respectively.

Let us consider the array in cylindrical coordinate system, the actual position of mth antenna element is (ρ_m, ξ_m, z_m), $m = 1 \ldots M$. With far-field assumption, the propagation delay in (3) can be written as

$$\tau_m(\Omega_p) = -\frac{1}{c}[\rho_m \sin\theta_p \cos(\xi_m - \phi_p) + z_m \cos\theta_p] \quad (4)$$

where c is the propagation velocity.

In the presence of model errors, the actual gains, phases, and positions will be given by

$$\gamma_m(\omega_k) = \bar{\gamma}_m(\omega_k) + \tilde{\gamma}_m(\omega_k)$$
$$\psi_m(\omega_k) = \bar{\psi}_m(\omega_k) + \tilde{\psi}_m(\omega_k) \quad (5)$$
$$\rho_m = \bar{\rho}_m + \tilde{\rho}_m;\ \xi_m = \bar{\xi}_m + \tilde{\xi}_m;\ z_m = \bar{z}_m + \tilde{z}_m \quad (6)$$

where $\bar{\gamma}_m(\omega_k)$, $\bar{\psi}_m(\omega_k)$, $\bar{\rho}_m$, $\bar{\xi}_m$ and $\bar{z}_m$ are the nominal (known) values and $\tilde{\gamma}_m(\omega_k)$, $\tilde{\psi}_m(\omega_k)$, $\tilde{\rho}_m$, $\tilde{\xi}_m$ and $\tilde{z}_m$ are their respective random errors.

B. Representation of Vector $\mathbf{a}(\omega_K, \omega_P)$

Applying the Bessel functions of the first kind to elements of $\mathbf{a}(\omega_k, \Omega_p)$ in (3), we obtain

$$\mathbf{a}(\omega_k, \Omega_p) \simeq [\mathbf{\Gamma}(\omega_k) \odot \Psi(\omega_k)] \{[\mathbf{B}_1(\omega_k)\mathbf{c}_1(\Omega_p)] \odot \odot [\mathbf{B}_2(\omega_k)\mathbf{c}_2(\Omega_p)] \odot [\mathbf{B}_3(\omega_k)\mathbf{c}_3(\theta_p)]\} \quad (7)$$

where $\odot$ is the Hadamard product [13]; $\mathbf{\Gamma}(\omega_k)$ and $\mathbf{\Psi}(\omega_k)$ are $(M \times M)$-diagonal matrices, $[\mathbf{\Gamma}(\omega_k)]_{mm} = \gamma_m(\omega_k)$, $[\mathbf{\Psi}(\omega_k)]_{mm} = e^{j\psi_m(\omega_k)}$; $\mathbf{B}_1(\omega_k)$, $\mathbf{B}_2(\omega_k)$, and $\mathbf{B}_3(\omega_k)$ are $(M \times (2n'+1))$-matrices containing the information of spatial frequency and array geometry, with

$$[\mathbf{B}_1(\omega_k)]_{mn} = [\mathbf{B}_2(\omega_k)]_{mn} = (j)^{2n} J_n\left(\frac{\omega_k}{2c}\rho_m\right) e^{-jn\xi_m}$$
$$[\mathbf{B}_3(\omega_k)]_{mn} = (j)^n J_n\left(\frac{\omega_k}{c} z_m\right). \quad (8)$$

$\mathbf{c}_1(\Omega_p)$, $\mathbf{c}_2(\Omega_p)$, $\mathbf{c}_3(\theta_p)$ are $((2n'+1)\times 1)$-vectors with $[\mathbf{c}_1(\Omega_p)]_n = e^{jn(\phi_p - \theta_p)}$, $[\mathbf{c}_2(\Omega_p)]_n = e^{jn(\phi_p + \theta_p)}$, $[\mathbf{c}_3(\theta_p)]_n = e^{jn(\theta_p)}$, $n = 0, \pm 1, \ldots, \pm n'$. In (8), $J_n(.)$ is the nth order Bessel function of the first kind, n' is the order of Bessel function selected such that $|J_n(.)|$ is small for $n > n'$.

By representing the vector $\mathbf{c}(\phi_p)$ using Taylor series as presented in [5], the relation between actual source-direction vector $\mathbf{a}(\omega_k, \phi_p)$ and source separation can be obtained.

III. Modified Nominal Transformation (MNT)

Let us denote the nominal (error-free) source-direction matrix at selected frequency ω_0 as

$$\bar{\mathbf{A}}(\omega_0) = [\bar{\mathbf{a}}(\omega_0, \Omega_1), \ldots, \bar{\mathbf{a}}(\omega_0, \Omega_p), \ldots, \bar{\mathbf{a}}(\omega_0, \Omega_P)]. \quad (9)$$

Representing the nominal source-direction vector at selected frequency ω_0, $\bar{\mathbf{a}}(\omega_0, \Omega_p)$, using Bessel functions, we obtain

$$\bar{\mathbf{a}}(\omega_0, \Omega_p) \simeq \left[\bar{\mathbf{\Gamma}}(\omega_0) \odot \bar{\Psi}(\omega_0)\right] \left\{\left[\bar{\mathbf{B}}_1(\omega_0)\mathbf{c}_1(\Omega_p)\right] \odot \odot \left[\bar{\mathbf{B}}_2(\omega_0)\mathbf{c}_2(\Omega_p)\right] \odot \left[\bar{\mathbf{B}}_3(\omega_0)\mathbf{c}_3(\theta_p)\right]\right\} \quad (10)$$

where $\bar{\mathbf{\Gamma}}(\omega_k)$ and $\bar{\mathbf{\Psi}}(\omega_k)$ are $(M \times M)$-diagonal matrices, with $\left[\bar{\mathbf{\Gamma}}(\omega_k)\right]_{mm} = \bar{\gamma}_m(\omega_k)$, $\left[\bar{\mathbf{\Psi}}(\omega_k)\right]_{mm} = e^{j\bar{\psi}_m(\omega_k)}$; and

$$\left[\bar{\mathbf{B}}_1(\omega_k)\right]_{mn} = \left[\bar{\mathbf{B}}_2(\omega_k)\right]_{mn} = (j)^{2n} J_n\left(\tfrac{\omega_k}{2c}\bar{\rho}_m\right) e^{-jn\bar{\xi}_m}$$
$$\left[\bar{\mathbf{B}}_3(\omega_k)\right]_{mn} = (j)^n J_n\left(\tfrac{\omega_k}{c}\bar{z}_m\right).$$

The transforming matrices $\mathbf{T}(\omega_k)$ at different frequencies ω_k, $\omega_l \le \omega_k \le \omega_h$, are determined by

$$\min_{\mathbf{T}(\omega_k)} \parallel \bar{\mathbf{A}}(\omega_0) - \mathbf{T}(\omega_k)\mathbf{A}(\omega_k) \parallel_F \quad (11)$$

where $\parallel . \parallel_F$ is the Frobenius matrix norm [14]. According to [9], the transforming is lossless if $\mathbf{T}(\omega_k)$ are unitary matrices. Therefore, (11) is subjected to $\mathbf{T}^\dagger(\omega_k)\mathbf{T}(\omega_k) = \mathbf{I}$, where the superscript † indicates Hermitean conjugate. The solution of (11) is then given by

$$\mathbf{T}(\omega_k) = \mathbf{L}_k \mathbf{R}_k^\dagger \quad (12)$$

where $\mathbf{L}_k$, $\mathbf{R}_k$ are $(M \times M)$-unitary matrices formed by left and right singular vectors of the product $\bar{\mathbf{A}}(\omega_0)\mathbf{A}^\dagger(\omega_k)$.

The transformed covariance matrix, $\mathbf{R}$, is obtained as

$$\mathbf{R} = \sum_{k=l}^{h} \mathbf{T}(\omega_k)\mathbf{R}(\omega_k)\mathbf{T}^\dagger(\omega_k) \quad (13)$$

where $\mathbf{R}(\omega_k) = E\{\mathbf{X}(\omega_k)\mathbf{X}^\dagger(\omega_k)\}$ is cross-spectral density matrix at ω_k. Defining a grid of I spatial frequency points, the estimated spectrum using MUSIC [15] is then given by

$$\hat{P}_{MNT-MUSIC} = \frac{\bar{\mathbf{a}}^\dagger(\omega_0, \Omega_i)\bar{\mathbf{a}}(\omega_0, \Omega_i)}{\bar{\mathbf{a}}^\dagger(\omega_0, \Omega_i)\, \mathbf{V}_N \mathbf{V}_N^\dagger \bar{\mathbf{a}}(\omega_0, \Omega_i)} \quad (14)$$

where $\mathbf{V}_N$ is noise-eigenvector matrix obtained from the covariance matrix $\mathbf{R}$, and $i = 1 \ldots I$. The DOA's can be then estimated by locating the highest peaks of $\hat{P}_{MNT-MUSIC}$.

IV. Simulation Results

We considers 1-D estimation of azimuth (elevation of $\pi/2$) using 16-element uniform circular array (UCA). The inter-element spacing is of $0.5\lambda_h$, with $\lambda_h = c/f_h$. Assumed that three signal sources, which have identical bandwidth of $0.5f_h$ spanned by 51 frequency bins, impinge on the array from directions 75°, 90°, and 105°. The simulations are performed with 500 independent trials and 51 snapshots are used. Assumed that the position error (PE), gain and phase errors (GPE) have Gaussian distribution with mean μ and standard deviation σ. For the PE, $\mu_{\tilde{\rho}} = 0$, $\sigma_{\tilde{\rho}} = 0.083$ ($\tilde{\rho}_m$ in range of $[-0.25\lambda_h, 0.25\lambda_h]$); $\mu_{\tilde{\xi}} = 0$, $\sigma_{\tilde{\xi}} = 1.67$ ($\tilde{\xi}_m$ in the range of [−5°, 5°]). The gain error is assumed with $\mu_{\tilde{\gamma}} = 0$, $\sigma_{\tilde{\gamma}} = 0.033$ ($\tilde{\gamma}_m$ in the range of [−0.1, 0]) and the phase error is assumed with $\mu_{\tilde{\psi}} = 0$, $\sigma_{\tilde{\psi}} = 1.67$ ($\tilde{\psi}_m$ in the range of [−5°, 5°]). We compare the performance of proposed method in conjunction with the MUSIC (MNT-MUSIC) and the Two-Sided Correlation Transformation method [10], which is considered with array errors (T-MUSIC-PE-GPE) and without errors (T-MUSIC). To evaluate the performance of methods, the bias that measures the average deviation of the estimator from the true value [17] and the standard deviation of estimate for different values of signal to noise ratio (SNR) are considered.

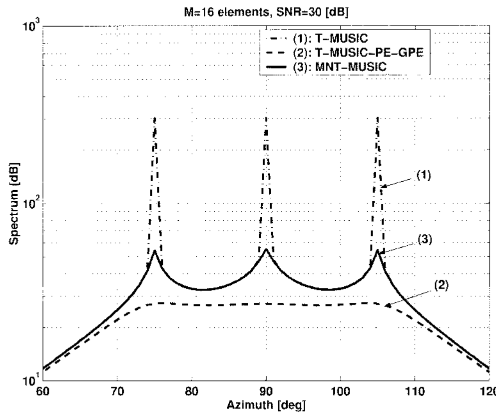

Fig. 1. Estimated spectra of T-MUSIC without error [the curve (1)], of T-MUSIC with errors [T-MUSIC-PE-GPE, curve (2)] and of the proposed method [MNT-MUSIC, curve (3)] for the sources at 75°, 90°, 105°.

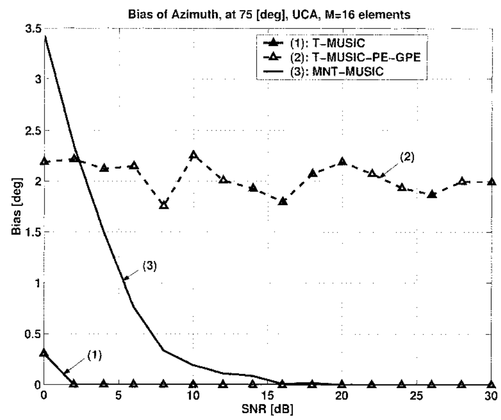

Fig. 2. Bias of estimate versus SNR at 75°.

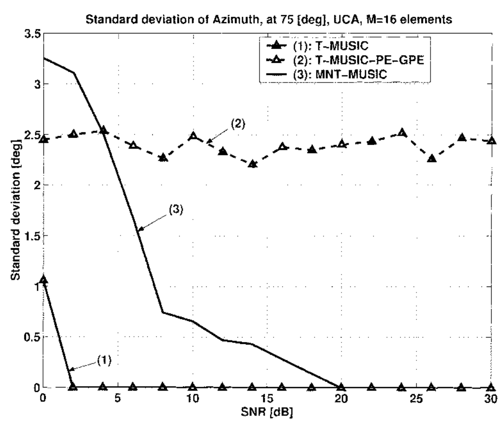

Fig. 3. Standard deviation of estimate versus SNR at 75°.

In Fig. 1, the estimated spatial spectra (versus azimuth angle) of the MNT-MUSIC (14), of the T-MUSIC without error (T-MUSIC) and of the T-MUSIC with error (T-MUSIC-PE-GPE) are overlaid for comparison. The bias and standard deviation of methods (versus SNR) are compared in Figs. 2 and 3, respectively. As illustrated in figures, the T-MUSIC, which provides unbias and low deviation at low SNR [16], is strongly degraded by errors. In the presence of errors, the bias and standard deviation cannot be improved with increasing SNR. As expected, the estimated spectrum of proposed method (MNT-MUSIC) is clearly improved in presence of the PE and the GPE (Fig. 1). As shown in Figs. 2 and 3, the performance of proposed method are improved as SNR increases, the proposed method is robust in the presence of array errors when the SNR is sufficiently high.

V. Conclusion

In this letter, a model of wideband DOA estimation for closely-spaced sources using arbitrary antenna array is presented. Practical considerations including the errors of array element positions and the error of element gain and phase are considered. To reduce these array errors, a wideband DOA estimation method without array calibration is proposed. The proposed method reduces the effects of errors at low SNR and is robust in the presence of errors at high SNR.

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